Sanjay Ghodawat University, Kolhapur

2020-21

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

EXM/P/09/00

Year and Program: S.Y. M.Sc. **School of Science** **Department : Mathematics**

Course Code: MTS609.1 **Course Title: Measure and Integration** **Semester : III**

Day : **End Semester Examination** **Time : 2 Hrs.**

Date : **Max Marks : 50**

Instructions: 1) All Questions are compulsory.
2) Figures to the right indicate full marks.

		Marks	Bloom's Level	CO
Q.1	**Answer any TWO of the following.**			
a)	What is open set? Show that any non-empty open set of real numbers can expressed as countable union of disjoint open intervals.	08	L2	CO1
b)	Show that for any $\epsilon > 0$ there is an open set $\vartheta \supset A$ with $m^*(\vartheta - A) < \in$, where A is a measurable set.	08	L2	CO1
c)	Define measurable function. Also investigate that for any constant α the function αf will be measurable function or not, provided f is measurable function on measurable set E.	08	L6	CO2
Q.2	**Answer any TWO of the following.**			
a)	State and prove Egoroff's theorem.	08	L3	CO2
b)	Show that $m\{x \in A: f(x) \geq \lambda\} \leq \frac{1}{\lambda}\int_A f$, where $\lambda > 0$ and f be a non-negative measurable function on the set A.	08	L3	CO3
c)	If $\phi \leq \psi$ almost everywhere on the set E then prove that $\int_E \phi \leq \int_E \psi$, where ϕ and ψ are simple functions which vanishes outside the set of finite measure E.	08	L4	CO3
Q.3	**Answer any TWO of the following.**			
a)	Is the monotone function f on the open interval (a, b) will be differentiable almost everywhere on (a, b)? Justify your answer.	09	L5	CO4

b)	i) Prove that the positive part and negative part of the measurable function f are integrable over the set E if and only if $\|f\|$ is integrable over E.	04	L3	CO4
	ii) State and prove Minkowski's inequality.	05	L1	CO5
c)	Define absolutely continuous function and prove that the absolutely continuous function f on a closed bounded interval $[a, b]$ is the difference of two increasing absolutely continuous functions.	09	L2	CO5

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